6.1 The Basics of Counting

6.1 pg 396 # 1
There are 18 mathematics majors and 325 computer science majors at a college.

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

We must first choose a math major. There are 18 ways to choose such a representative. Now we must choose a CS major. There are 325 ways to choose such a representative. Since we have two sequential tasks, we use the product rule. Therefore, there are $18 \times 325 = 5,850$ ways.

6.1 pg 396 # 3
A multiple-choice test contains 10 questions. There are four possible answers for each question.

a) In how many ways can a student answer the questions on the test if the student answers every question?

Here, there are 4 choices per question, and 10 questions. Choosing an answer for each question is a sequential task, so there are $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^{10}$ ways to answer.

b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

There are now 5 choices per question, but the process is the same as in problem a). Therefore, there are $5^{10}$ ways to answer.

6.1 pg 396 # 7
How many different three-letter initials can people have?

One has 26 choices for the first initial, 26 for the second, and 26 for the third. Therefore, there are $26^3$ possible three-letter initials.

6.1 pg 396 # 9
How many different three-letter initials are there that begin with an A?

While this question is asking about 3 letter initials, the first letter is already chosen for us, which is A. Therefore we only need to worry about the other two letters. There are 26 letters to choose from, and two sequential choices to make. So, there are $26^2$ possible initials.
6.1 pg 396 # 11

How many bit strings of length ten both begin and end with a 1?

This is similar to the above question (problem #9). We are asked about 10 bits, but the first and the last bits are already chosen for us. Bits are either 1 or 0, so there are 2 choices per bit, and 8 bits to choose. Therefore, there are $2^8 = 256$ bit strings of length ten that begin and end with a 1.

6.1 pg 397 # 25

How many strings of three decimal digits

a) do not contain the same digit three times?

There are $10^3 = 1000$ decimal strings with 3 digits, and there are 10 of them containing three equal digits: 000, 111, ..., 999. Therefore, there are $1000 - 10 = 990$ strings that do not contain the same digit three times.

b) begin with an odd digit?

There are 5 odd digits, therefore we have $5 \times 10 \times 10 = 500$ strings of three decimal digits beginning with an odd digit.