1.6 Rules of Inference

An *Inference Rule* is a pattern establishing that if we know that a set of premise statements of certain forms are all true, then we can validly deduce that a certain related conclusion statement is true.

Inference Rules

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\dfrac{\forall x P(x)}{\therefore P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$	Existential generalization

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What rules of inference are used in this famous argument? "All men are mortal. Socrates is a man. Therefore, Socrates is mortal."
$M(x)$ = "x is mortal"
$N(x)$ = "x is a man"

Premise 1	"All men are mortal."	$\forall x(N(x) \rightarrow M(x))$
Premise 2	"Socrates is a man."	$N(\text{Socrates})$
Conclude	"Socrates is mortal."	$M(\text{Socrates})$

Step		**Reason**
1	$\forall x(N(x) \rightarrow M(x))$	Premise 1
2	$N(\text{Socrates}) \rightarrow M(\text{Socrates})$	Universal Instantiation from (1)
3	$N(\text{Socrates})$	Premise 2
$\therefore$	$M(\text{Socrates})$	Modus Ponens from (2) and (3)

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For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

b "If I eat spicy foods, then I have strange dreams." "I have strange dreams if there is thunder while I sleep." "I did not have strange dreams."

S = "I ate spicy food"
D = "I had strange dreams"
T = "It thundered while I slept"

"If I eat spicy foods, then I have strange dreams."	$S \rightarrow D$
"I have strange dreams if there is thunder while I sleep."	$T \rightarrow D$
"I did not have strange dreams."	$\neg D$

Step		**Reason**	
1	$S \to D$	Premise 1	
2	$T \to D$	Premise 2	
3	$\neg D$	Premise 3	
4	$\neg T$	Modus Tollens from (2) and (3)	"It did not thunder while I slept."
5	$\neg S$	Modus Tollens from (1) and (3)	"I did not eat spicy food"
6	$\neg T \land \neg S$	Conjunction from (4) and (5)	"It did not thunder and I did not eat spicy food"

a "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."

$O(x)$ = "I took x off"
$R(x)$ = "it rains on x"
$S(x)$ = "it snows on x"

"If I take the day off, it either rains or snows."	$\forall x(O(x) \to (R(x) \lor S(x)))$
"I took Tuesday off or I took Thursday off."	$O(\text{Tue}) \lor O(\text{Thu})$
"It was sunny on Tuesday."	$\neg S(\text{Tue}) \land \neg R(\text{Tue})$
"It did not snow on Thursday."	$\neg S(\text{Thu})$

Step		**Reason**	
1	$\forall x(O(x) \to (R(x) \lor S(x)))$	Premise 1	
2	$\neg S(\text{Tue}) \land \neg R(\text{Tue})$	Premise 3	
3	$O(\text{Tue}) \to (R(\text{Tue}) \lor S(\text{Tue}))$	Universal Instantiation from (1) using "Tue"	
4	$\neg(R(\text{Tue}) \lor S(\text{Tue}))$	De Morgan's Law from (2)	
5	$\neg O(\text{Tue})$	Modus Tollens from (3) and (4)	"I did not take Tues off."
6	$O(\text{Tue}) \lor O(\text{Thu})$	Premise 2	
7	$O(\text{Thu})$	Disjunctive Syllogism from (5) and (6)	"I took Thu off."
8	$O(\text{Thu}) \to (R(\text{Thu}) \lor S(\text{Thu}))$	Universal Instantiation from (1) using "Thu"	
9	$R(\text{Thu}) \lor S(\text{Thu})$	Modus Ponens from (7) and (8)	"Either it rained or snowed on Thu."
10	$\neg S(\text{Thu})$	Premise 4	
11	$R(\text{Thu})$	Disjunctive syllogism from (9) and (10)	"It rained Thu."

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For each of these arguments, explain which rules of inference are used for each step.

a "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."

$C(x)$ = "x is in this class"
$J(x)$ = "x knows how to write programs in JAVA"
$H(x)$ = "x can get a high paying job"

Premise 1	C(Doug)
Premise 2	J(Doug)
Premise 3	$\forall x(J(x) \to H(x))$
Conclude	$\exists x(C(x) \land H(x))$

Step		**Reason**
1	$\forall x(J(x) \to H(x))$	Premise 3
2	J(Doug) $\to H$(Doug)	Universal Instantiation from (1)
3	J(Doug)	Premise 2
4	H(Doug)	Modus Ponens from (2) and (3)
5	C(Doug)	Premise 1
6	C(Doug)$\land H$(Doug)	Conjunction from (4) and (5)
$\therefore$	$\exists x(C(x) \land H(x))$	Existential generalization from (6)

c "Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program."

$C(x)$ = "x is in this class"
$P(x)$ = "x owns a personal computer"
$W(x)$ = "x can use a word processing program"

Premise 1	$\forall x(C(x) \to P(x))$
Premise 2	$\forall x(P(x) \to W(x))$
Premise 3	C(Zeke)
Conclude	W(Zeke)

Step		**Reason**
1	$\forall x(C(x) \to P(x))$	Premise 1
2	C(Zeke)$\to P$(Zeke)	Universal Instantiation on (1)
3	C(Zeke)	Premise 3
4	P(Zeke)	Modus Ponens on (2) and (3)
5	$\forall x(P(x) \to W(x))$	Premise 2
6	P(Zeke)$\to W$(Zeke)	Universal Instantiation on (5)
$\therefore$	W(Zeke)	Modus Ponens on (4) and (6)

d "Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean."

$J(x)$ = "x lives in New Jersey"
$O(x)$ = "x lives within 50 miles of the ocean"

$S(x)$ = "x has seen the ocean"

Premise 1	$\forall x(J(x) \rightarrow O(x))$
Premise 2	$\exists x(J(x) \wedge \neg S(x))$
Conclude	$\exists x(O(x) \wedge \neg S(x))$

Step		**Reason**
1	$\exists x(J(x) \wedge \neg S(x))$	Premise 2
2	$J(y) \wedge \neg S(y)$	Existential Instantiation on (1) (y is an element of the domain)
3	$J(y)$	Simplification on (2)
4	$\forall x(J(x) \rightarrow O(x))$	Premise 1
5	$J(y) \rightarrow O(y)$	Universal Instantiation on (4)
6	$O(y)$	Modus Ponens on (3) and (5)
7	$\neg S(y)$	Simplification on (2)
8	$O(y) \wedge \neg S(y)$	Conjunction on (6) and (7)
$\therefore$	$\exists x(O(x) \wedge \neg S(x))$	Existential generalizing on (8)