5.3 Recursive Definitions

5.3 pg 357 # 1
Find \( f(1), f(2), f(3), \) and, \( f(4) \) if \( f(n) \) is defined recursively by \( f(0) = 1 \) and for \( n = 0, 1, 2, \ldots \)

a) \( f(n + 1) = f(n) + 2 \)

b) \( f(n + 1) = 3f(n) \)

5.3 pg 358 # 7
Give a recursive definition of the sequence \( \{a_n\}, n = 1, 2, 3, \ldots \) if

a) \( a_n = 6n \)

b) \( a_n = 2n + 1 \)

5.3 pg 358 # 25
Give a recursive definition of

a) the set of even integers.

b) the set of positive integers congruent to 2 modulo 3.

c) the set of positive integers not divisible by 5.

5.3 pg 358 # 27
Let \( S \) be the subset of the set of ordered pairs of integers defined recursively by

- Basis Step: \((0, 0) \in S\)

- Recursive Step: If \((a, b) \in S\), then \((a, b+1) \in S\), \((a+1, b+1) \in S\), and \((a+2, b+1) \in S\).

a) List the elements of \( S \) produced by the first four applications of the recursive definition.

c) Use structural induction to show that \( a \leq 2b \) whenever \((a, b) \in S\).

5.3 pg 359 # 37
Give a recursive definition of \( w^i \), where \( w \) is a string and \( i \) is a nonnegative integer. (Here \( w^i \) represents the concatenation of \( i \) copies of the string \( w \).)