2.1 Sets

A set is an unordered collection of objects, called elements or members of the set. A set is said to contain its elements.

We write \( a \in A \) to denote that \( a \) is an element of the set \( A \). The notation \( a \notin A \) denotes that \( a \) is not an element of the set \( A \).

Two sets are equal if and only if they have the same elements. We write \( A = B \) if \( A \) and \( B \) are equal sets.

Empty Set

The empty set or null set is the set with no elements. Denoted by \( \emptyset \) or \{ \}.

Other Special Sets

\( \mathbb{N} = \{0, 1, 2, 3, \ldots\} \), the set of natural numbers
\( \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\} \), the set of integers
\( \mathbb{Z}^+ = \{1, 2, 3, \ldots\} \), the set of positive integers
\( \mathbb{Q} = \{p/q | p \in \mathbb{Z}, q \in \mathbb{Z}, \text{ and } q \neq 0\} \), the set of rational numbers
\( \mathbb{R} \), the set of real numbers
\( \mathbb{R}^+ \), the set of positive real numbers
\( \mathbb{C} \), the set of complex numbers.

Subset

The set \( A \) is a subset of \( B \) if and only if every element of \( A \) is also an element of \( B \). We use the notation \( A \subseteq B \) to indicate that \( A \) is a subset of the set \( B \).

To show that \( A \subsetneq B \), find a single \( x \in A \) such that \( x \notin B \).

Note that \( \emptyset \) is the subset of every set.

Proper Subset

To show that \( A \) is a subset of \( B \) and \( A \neq B \), we use \( A \subset B \) to denote proper subset. \( A \subset B \) says that \( A \) is a proper subset of \( B \).

Cardinality

The cardinality of a set is the number of distinct elements within the set. The cardinality of set \( A \) is \( |A| \). Note that \( |\emptyset| = 0 \).
Power Sets

The set of all subsets of a set \( A \), denoted \( \rho(A) \), is called the power set of \( A \).

Cartesian Product

The Cartesian Product of two sets \( A \) and \( B \), denoted by \( A \times B \), is the set of ordered pairs \( (a, b) \) where \( a \in A \) and \( b \in B \).

Note that \( A \times B \neq B \times A \).

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List the members of these sets.

\[ \{ x \mid x \text{ is the square of an integer and } x < 100 \} \]
\[ \{ 0, 1, 4, 9, 16, 25, 36, 49, 64, 81 \} \]

\[ \{ x \mid x \text{ is an integer such that } x^2 = 2 \} \]
\[ \emptyset \text{ or } \{ \} \]

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Determine whether each pairs of sets are equal.

\[ \{ 1, 3, 3, 3, 5, 5, 5, 5, 5 \}, \{ 5, 3, 1 \} \]
Yes

\[ \{ \{ 1 \} \}, \{ 1, \{ 1 \} \} \]
No

\[ \emptyset, \{ \emptyset \} \]
No

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Determine whether each of these statements is true or false.

\[ 0 \in \emptyset \]
False

\[ \emptyset \in \{ 0 \} \]
False

\[ \emptyset \subset \{ 0 \} \]
True
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Determine whether each of these statements is true or false.

a \( x \in \{x\} \)
   True

b \( \{x\} \subseteq \{x\} \)
   True

c \( \{x\} \in \{x\} \)
   False

d \( \{x\} \in \{\{x\}\} \)
   True

e \( \emptyset \subseteq \{x\} \)
   True

f \( \emptyset \in \{x\} \)
   False

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What is the cardinality of each of these sets?

b \( \{\{a\}\} \)
   1

c \( \{a, \{a\}\} \)
   2

d \( \{a, \{a\}, \{a, \{a\}\}\} \)
   3
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Find the power set of each of these sets, where \( a \) and \( b \) are distinct elements.

\[
\begin{align*}
a &\quad \{a\} \\
&\quad \{\emptyset, \{a\}\} \\
b &\quad \{a, b\} \\
&\quad \{\emptyset, \{a\}, \{b\}, \{a, b\}\}
\end{align*}
\]

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Explain why \( A \times B \times C \) and \( (A \times B) \times C \) are not the same.

First, \( A \times B \times C \) consists of 3-tuples \((a, b, c)\), where \( a \in A \), \( b \in B \), and \( c \in C \). Next, \((A \times B) \times C\) contains the elements \([(a, b), c]\), which is a set of ordered pairs with one of them being an ordered pair. An ordered pair and a 3-tuple are two different collections.