12.2 Representing Boolean Functions

Literals

A *literal* is a Boolean variable or its complement.

Minterm

A *minterm* of the Boolean variables $x_1, x_2, \ldots, x_n$ is a Boolean product $y_1 y_2 \ldots y_n$, where $y_i = x_i$ or $y_i = \bar{x}_i$. Hence, a minterm is a product of n literals, with one literal for each variable.

Disjunctive Normal Form

The *disjunctive normal form* (DNF) of a *degree-n* Boolean function f is the unique sum of minterms of the variables $x_1, \ldots, x_n$ that represents f.

Maxterm

A *maxterm* of the Boolean variables $x_1, x_2, \ldots, x_n$ is a Boolean sum $y_1 + y_2 + \ldots + y_n$, where $y_i = x_i$ or $y_i = \bar{x}_i$. Hence, a maxterm is a sum of n literals, with one literal for each variable.

Conjunctive Normal Form

The *conjunctive normal form* (CNF) of a *degree-n* Boolean function f is the unique product of maxterms of the variables $x_1, \ldots, x_n$ that represents f.

Functional Completeness

Since every Boolean function can be expressed in terms of $\cdot, +, ^{-}$, we say that the set of operators $\{\cdot, +, ^{-}\}$ is *functionally complete*. $\{+, ^{-}\}$ and $\{\cdot, ^{-}\}$ are also functionally complete. $\{\cdot, ^{-}\}$ can be represented by the NAND operator $|$ and $\{+, ^{-}\}$ by the NOR operator $\downarrow$. There fore $\{|\}$ and $\{\downarrow\}$ are also functionally complete.

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Find a Boolean product of the Boolean variables $x, y,$, and z, or their complements, that has the value 1 if and only if

a) $x = y = 0, z = 1$

   Since $x = y = 0$, then $\bar{x} = \bar{y} = 1$. The boolean product is simply $\bar{x}\bar{y}z$.

b) $x = 0, y = 1, z = 0$

   $\bar{x}y\bar{z}$

c) $x = 0, y = z = 1$

   $\bar{x}yz$

d) $x = y = z = 0$

$\bar{x}\bar{y}\bar{z}$

12.2 pg. 822 # 3

Find the sum-of-products expansions of these Boolean functions.

a) $F(x, y, z) = x + y + z$

Create the table for the function and take the Boolean sum of the minterm where the function evaluates to 1.

x	y	z	$x+y+z$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

The DNF is $\bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z} + xyz$

b) $F(x, y, z) = (x + z)y$

x	y	z	$x+z$	$(x+z)y$
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1

The DNF is $\bar{x}yz + xy\bar{z} + xyz$

c) $F(x, y, z) = x$

x	y	z
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

The DNF is $x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z} + xyz$

d) $F(x, y, z) = x\bar{y}$

x	y	z	$\bar{y}$	$x\bar{y}$
0	0	0	1	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0

The DNF is $x\bar{y}\bar{z} + x\bar{y}z$.

12.2 pg. 822 # 5

Find the sum-of-products expansion of the Boolean function $F(w, x, y, z)$ that has the value 1 if and only if an odd number of w, x, y, and z have the value 1.

Need to produce all the minterms that have an odd number of 1s. The DNF is simply,

$$\bar{w}xyz + w\bar{x}yz + wx\bar{y}z + wxy\bar{z} + \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}y\bar{z} + \bar{w}x\bar{y}\bar{z} + w\bar{x}\bar{y}\bar{z}$$

12.2 pg. 822 # 11

Find the product-of-sums expansion of each of the Boolean functions in Exercise 3.

a) $F(x, y, z) = x + y + z$

Create the table for the function and take the Boolean product of the maxterm where the function evaluates to 0.

x	y	z	$x + y + z$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

This is already in its CNF, $x + y + z$.

b) $F(x, y, z) = (x + z)y$

x	y	z	$x+z$	$(x+z)y$
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1

The CNF is $(x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})$

c) $F(x,y,z) = x$

x	y	z
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

The CNF is $(x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(x+\bar{y}+\bar{z})$

d) $F(x,y,z) = x\bar{y}$

x	y	z	$\bar{y}$	$x\bar{y}$
0	0	0	1	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0

The CNF is $(x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(x+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})$

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Express each of these Boolean functions using the operators $+$ and $^-$.

a) $x+y+z$

$x+y+z$ already satisfies the requirement.

b) $x+\bar{y}(\bar{x}+z)$

$x + \bar{y}(\bar{x} + z)$
$= x + \bar{y}\bar{x} + \bar{y}z$ by distributive law
$= x + \overline{(y + x)} + \bar{y}z$ by De Morgan's law
$= x + \overline{(y + x)} + \overline{(y + \bar{z})}$ by De Morgan's law

c) $\overline{x + \bar{y}}$

$\overline{x + \bar{y}}$ already satisfies the requirement.

d) $\bar{x}(x + \bar{y} + \bar{z})$

$\bar{x}(x + \bar{y} + \bar{z})$
$= (\bar{x}x) + (\bar{x}\bar{y}) + (\bar{x}\bar{z})$ by distributive law
$= 0 + (\bar{x}\bar{y}) + (\bar{x}\bar{z})$ by zero property
$= (\bar{x}\bar{y}) + (\bar{x}\bar{z})$ by identity law
$= \overline{(x + y)} + (\bar{x}\bar{z})$ by De Morgan's law
$= \overline{(x + y)} + \overline{(x + z)}$ by De Morgan's law