13.3 Finite-State Machines with No Output

Concatenation

Suppose that $A$ and $B$ are subsets of $V^*$, where $V$ is a vocabulary. The concatenation of $A$ and $B$, denoted by $AB$, is the set of all strings of the form $xy$, where $x$ is a string in $A$ and $y$ is a string in $B$.

Kleene closure

Suppose that $A$ is a subset of $V^*$. Then the Kleene closure of $A$, denoted by $A^*$, is the set consisting of concatenations of arbitrarily many strings from $A$. That is, $A^* = \bigcup_{k=0}^{\infty} A^k$.

Finite-state Automata

*Finite-state automata* are finite-state machines with no output. A finite-state automaton $M = (S, I, f, s_0, F)$ consists of

- a finite set $S$ of states
- a finite input alphabet $I$
- a transition function $f : S \times I \rightarrow S$
- an initial state $s_0$
- a finite set $F$ of final states (or accepting states)

![Finite-state automaton diagram]

<table>
<thead>
<tr>
<th>State</th>
<th>Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
<td>0</td>
</tr>
<tr>
<td>$s_1$</td>
<td>$s_0$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>$s_0$</td>
</tr>
<tr>
<td></td>
<td>$s_2$</td>
</tr>
</tbody>
</table>

Language Recognition by Finite-State Machines

A string $x$ is said to be recognized or accepted by the machine $M = (S, I, f, s_0, F)$ if it takes the initial state $s_0$ to a final state, that is $f(s_0, x)$ is a state in $F$. The language recognized or accepted by the machine $M$, denoted by $L(M)$, is the set of all strings that are recognized by $M$. Two finite-state automata are called equivalent if they recognize the same language.

Nondeterministic Finite State Automata

So far we have only discussed deterministic finite state automata because each pair of state and input value has a unique next state given by the transition function. We will now discuss nondeterministic finite state automata where there can be several possible next states for each pair of state and input value.

A nondeterministic finite-state automaton $M = (S, f, s_0, F)$ consists of
• a finite set \( S \) of states
• a finite input alphabet \( I \)
• a transition function \( f \) \((f : S \times I \rightarrow P(S))\)
• an initial state \( s_0 \)
• a finite set \( F \) of final states

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Theorem 1

If the language \( L \) is recognized by a nondeterministic finite-state automaton \( M_0 \), then \( L \) is also recognized by a deterministic finite-state automaton \( M_1 \).

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Let \( A = \{0, 11\} \) and \( B = \{00, 01\} \). Find each of these sets.

a) \( AB \)
\[ AB = \{000, 001, 1100, 1101\} \]
b) \( BA \)
\[ BA = \{000, 0011, 010, 0111\} \]
c) \( A^2 \)
\[ A^2 = \{00, 011, 110, 1111\} \]

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Describe the elements of the set \( A^* \) for these values of \( A \).

a) \( \{10\} \)
The set of strings where there are zero or more copies of 10, defined as \( \{(10)^n | n = 0, 1, 2, \ldots \} \)
b) \( \{111\} \)
The set of strings where there are zero or more copies of 111, defined as \( \{1^{3n} | n = 0, 1, 2, \ldots \} \)
c) \( \{0, 01\} \)
The set of strings where every 1 is immediately preceded by a 0.
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Determine whether the string 11101 is in each of these sets.

a) \{0, 1\}*
   Yes.

b) \{1\}^*\{0\}^*\{1\}^*
   Yes.

c) \{11\}\{0\}^*\{01\}
   No.

d) \{11\}^*\{01\}^*
   No.

e) \{111\}^*\{0\}^*\{1\}
   Yes.

f) \{11, 0\}\{00, 101\}
   Yes.

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Find the language recognized by the given deterministic finite-state automaton.

![DFA Diagram](image)

The language is \{0, 10, 11\}\{0, 1\}^*

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Find the language recognized by the given deterministic finite-state automaton.

![DFA Diagram](image)

The language is \{0\}^*\{1\}\{1\}^*
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Construct a deterministic finite-state automaton that recognizes the set of all bit strings beginning with 01.

![Diagram of an automaton recognizing strings beginning with 01]

13.3 pg. 876 # 27
Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain exactly three 0s.

![Diagram of an automaton recognizing strings with exactly three 0s]

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Find the language recognized by the given nondeterministic finite-state automaton.

![Diagram of a nondeterministic automaton]

Since the initial state is an accepting state, we know that the automaton accepts the empty string, $\lambda$. We now need to figure how to enter the second accepting state, $s_2$. By inspection, we can see that we have two ways to reach there, by $s_0$ or by going through $s_1$. Let us first consider going through $s_1$. We can only reach $s_2$ by inputing one or more 0s followed by one or more 1s. The other way to reach $s_2$ is to skip $s_1$ by inputing one 0 and zero or more 1s. Thus, the language recognized is $\{\lambda\} \cup \{0^n1^m | n, m \geq 1\} \cup \{01^m | m \geq 0\}$. 