8.5 Inclusion-Exclusion

Theorem 1: Let $A_1, A_2, ..., A_n$ be finite sets. Then:

$$|A_1 \cup A_2 \cup ... \cup A_n| = \sum_{1 \le i \le n} |A_i| - \sum_{1 \le i < j \le n} |A_i \cap A_j| + \sum_{1 \le i < j < k \le n} |A_i \cap A_j \cap A_k| - ... + (-1)^{n+1} |A_1 \cap A_2 \cap ... \cap A_n|$$

8.5 pg. 557 # 3

A survey of households in the United States reveals that 96% have at least one television set, 98% have telephone service, and 95% have telephone service and at least one television set. What percentage of households in the United States have neither telephone service nor a television set?

Let V denote the set of households with television sets, and P denote the set of households with telephone service. From the problem statement, we know $|V| = 96$, $|P| = 98$, and $|V \cap P| = 95$. By inclusion-exclusion, we know that $|V \cup P| = |V| + |P| - |V \cap P| = 96 + 98 - 95 = 99$. Since we're looking for the set of households without telephone service nor television sets, then $100 - |V \cup P| = 100 - 99 = 1$. Only 1% of households have neither telephone service nor a television set.

8.5 pg. 557 # 7

There are 2504 computer science students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Linux, and 345 have taken a course in C. Further, 876 have taken courses in both Java and Linux, 231 have taken courses in both Linux and C, and 290 have taken courses in both Java and C. If 189 of these students have taken courses in Linux, Java, and C, how many of these 2504 students have not taken a course in any of these three programming languages?

Let J denote the set of students that have taken a course in Java, L denote the set of students that have taken a course in Linux, and C denote the set of students that have taken a course in C. We know that $|J| = 1876, |L| = 999, |C| = 345, |J \cap L| = 876, |L \cap C| = 231, |J \cap C| = 290, |J \cap L \cap C| = 189$. By inclusion-exclusion:
$|J \cup L \cup C| = |J| + |L| + |C| - |J \cap L| - |L \cap C| - |J \cap C| + |J \cap L \cap C|$
$|J \cup L \cup C| = 1876 + 999 + 345 - 876 - 231 - 290 + 189$
$|J \cup L \cup C| = 2012$

We have to find $2504 - |J \cup L \cup C| = 2504 - 2012 = 492$. 492 students have not taken course in any of these three programming languages.

8.5 pg. 558 # 11

Find the number of positive integer not exceeding 100 that are either odd or the square of an integer.

Let O denote the set of odd numbers not exceeding 100 and S denote the set of the squares of an integer not exceeding 100. We know $|O| = 50$ because there are 50 odd integers not exceeding 100.

We know $|S| = 10$ because there are only 10 numbers that are squares of integers not exceeding 100 (1^2 to 10^2). Of these squares, we know 5 of them are odd (1, 9, 25, 49, 81), so $|O \cap S|$. We are looking for $|O \cup S|$, which is just $|O| + |S| - |O \cap S|$. So, $|O \cup S| = 50 + 10 - 5 = 55$.