8.1 Applications of Recurrence Relations

8.1 pg. 510 # 3
A vending machine dispensing books of stamps accepts only one-dollar coins, $1 bills, and $5 bills.

a Find a recurrence relation for the number of ways to deposit \( n \) dollars in the vending machine, where the order in which the coins and bills are deposited matters.

b What are the initial conditions?

c How many ways are there to deposit $10 for a book of stamps?

8.1 pg. 510 # 7

a Find a recurrence relation for the number of bit strings of length \( n \) that contain a pair of consecutive 0s.

b What are the initial conditions?

c How many bit strings of length seven contain two consecutive 0s?

8.1 pg. 511 # 19

Messages are transmitted over a communications channel using two signals. The transmittal of one signal requires 1 microsecond, and the transmittal of the other signal requires 2 microseconds.

a Find a recurrence relation for the number of different messages consisting of sequences of these two signals, where each signal in the message is immediately followed by the next signal, that can be sent in \( n \) microseconds.

b What are the initial conditions?

c How many different messages can be sent in 10 microseconds using these two signals?

8.1 pg. 512 # 27

a Find a recurrence relation for the number of ways to lay out a walkway with slate tiles if the tiles are red, green, or gray, so that no two red tiles are adjacent and tiles of the same color are considered indistinguishable.

b What are the initial conditions for the recurrence relation in part (a)?

c How many ways are there to lay out a path of seven tiles as described in part (a)?