10.8 Graph Coloring

A *coloring* of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

Chromatic number

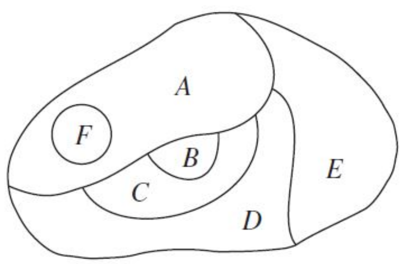
The *chromatic number* of a graph is the least number of colors needed for a coloring of this graph.

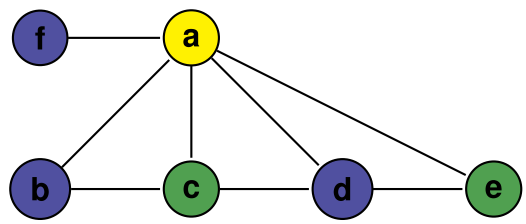
The Four Color Theorem

The chromatic number of a planar graph is no greater than four.

10.8 pg. 733 # 3

Construct the dual graph for the map shown. Then find the number of colors needed to color the map so that no two adjacent regions have the same color.

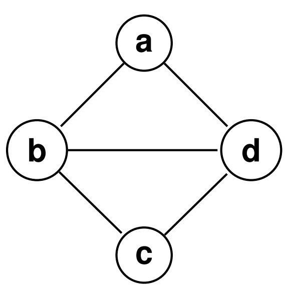




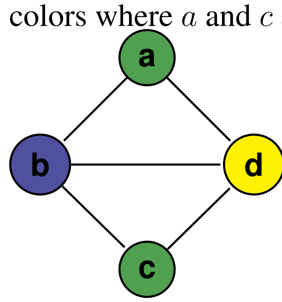
At least three colors are needed to color the graph because of triangle $\triangle abc$ exists in the graph.

10.8 pg. 733 # 7

Find the chromatic number of the given graph.

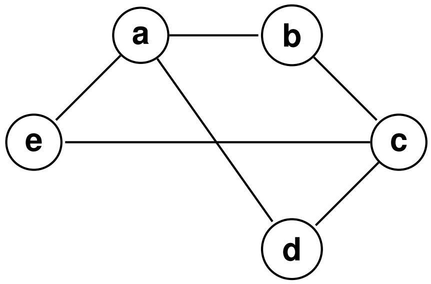


Since this graph forms two triangles, $\triangle abd$ and $\triangle bcd$, we can color this graph with at least 3 colors where a and c are the same colors.

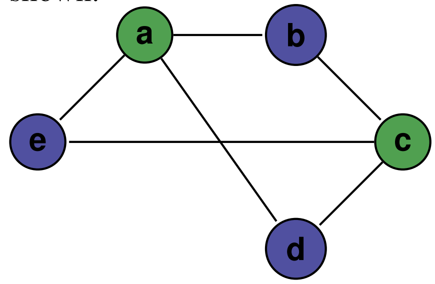


10.8 pg. 733 # 9

Find the chromatic number of the given graph.



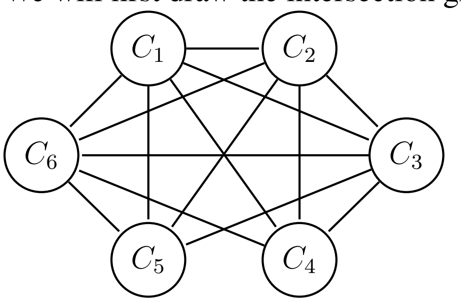
This graph can be colored with two colors like shown.



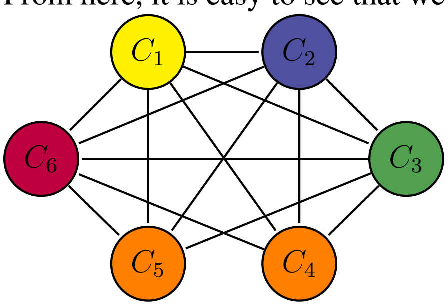
10.8 pg. 734 # 19

The mathematics department has six committees, each meeting once a month. How many different meeting times must be used to ensure that no member is scheduled to attend two meetings at the same time if the committees are C_1 = {Arlinghaus, Brand, Zaslavsky}, C_2 = {Brand, Lee, Rosen}, C_3 = {Arlinghaus, Rosen, Zaslavsky}, C_4 = {Lee, Rosen, Zaslavsky}, C_5 = {Arlinghaus, Brand}, and C_6 = {Brand, Rosen, Zaslavsky}?

We will first draw the intersection graph of the given sets.



From here, it is easy to see that we need at least 5 colors like so:



Therefore, 5 meeting times are needed. Committees C_4 and C_5 can meet at the same time.