10.1 Graphs and Graph Models

A graph \( G = (V, E) \) consists of \( V \), a nonempty set of vertices (or nodes) and \( E \), a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.

**Simple Graph**

A simple graph \( G = (V, E) \) consists of a set \( V \) of vertices or nodes and a set \( E \) of edges (or arcs or links) connecting unordered pairs of distinct elements \( u, v \in V \).

- No arrows
- No loops
- Can’t have multiple edges joining vertices

![Simple Graph Example](image)

**MultiGraph**

Like simple graphs, but there may be more than one edge connecting two given nodes.

![MultiGraph Example](image)

**Pseudographs**

Like multigraphs, but edges connecting a node to itself are allowed (i.e. loops are allowed).

![Pseudograph Example](image)
Simple Directed Graph

A simple directed graph $G = (V, E)$ consists of a set of vertices $V$ and a set of edges $E$ that are ordered pairs of elements of $V$.

- Directed edges
- No loops
- No multiple directed edges (i.e. at most one edge can be associated to each ordered pair of vertices)

\[ \begin{array}{ccc}
  a & \rightarrow & b \\
  \downarrow & & \downarrow \\
  d & \rightarrow & c \\
\end{array} \]

Directed Multigraph

Like a simple directed graph but there may be more than one edge connecting two given nodes and loops are allowed.

\[ \begin{array}{ccc}
  a & \leftrightarrow & b \\
  \downarrow & & \downarrow \\
  d & \leftrightarrow & c \\
\end{array} \]

Types of Graphs

<table>
<thead>
<tr>
<th>Type</th>
<th>Edges</th>
<th>Multiple Edges Allowed?</th>
<th>Loops Allowed?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple graph</td>
<td>Undirected</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Multigraph</td>
<td>Undirected</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Pseudograph</td>
<td>Undirected</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Simple directed graph</td>
<td>Directed</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Directed multigraph</td>
<td>Directed</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Mixed graph</td>
<td>Directed and undirected</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

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Construct a niche overlap graph for six species of birds, where the hermit thrush competes with the robin and with the blue jay, the robin also competes with the mockingbird, the mockingbird also competes with the blue jay, and the nuthatch competes with the hairy woodpecker.
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Construct an influence graph for the board members of a company if the President can influence the Director of Research and Development, the Director of Marketing, and the Director of Operations; the Director of Research and Development can influence the Director of Operations; the Director of Marketing can influence the Director of Operations; and no one can influence, or be influenced by, the Chief Financial Officer.