9.6 Partial Orderings

9.6 pg. 630 # 1
Which of these relations on \{0, 1, 2, 3\} are partial orderings? Determine the properties of a partial ordering that the others lack.

a) \{(0, 0), (1, 1), (2, 2), (3, 3)\}

b) \{(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}

c) \{(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)\}

d) \{(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}

e) \{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}

9.6 pg. 630 # 3
Is \((S, R)\) a poset if \(S\) is the set of all people in the world and \((a, b) \in R\), where \(a\) and \(b\) are people, if

a) \(a\) is taller than \(b\)?

b) \(a\) is not taller than \(b\)?

c) \(a = b\) or \(a\) is an ancestor of \(b\)?

d) \(a\) and \(b\) have a common friend?

9.6 pg. 630 # 5
Which of these are posets?

a) \((\mathbb{Z}, =)\)

b) \((\mathbb{Z}, \neq)\)

c) \((\mathbb{Z}, \geq)\)

d) \((\mathbb{Z}, \mid)\)

9.6 pg. 630 # 11
Determine whether the relation with the directed graph shown is a partial order.

\[
\begin{array}{c}
\text{a} \\
\rightarrow
\end{array}
\begin{array}{c}
\text{b} \\
\leftarrow
\end{array}
\]
9.6 pg. 630 # 19

Find the lexicographic ordering of the bit strings 0, 01, 11, 001, 010, 011, 0001, and 0101 based on the ordering 0 < 1.

9.6 pg. 631 # 23

Draw the Hasse diagram for divisibility on the set

a) \{1, 2, 3, 4, 5, 6, 7, 8\}

b) \{1, 2, 3, 5, 7, 11, 13\}

c) \{1, 2, 3, 6, 12, 24, 36, 48\}

9.6 pg. 631 # 33

Answer these questions for the poset \( (\{3, 5, 9, 15, 24, 45\}, \mid) \).

a) Find the maximal elements.

b) Find the minimal elements.

c) Is there a greatest element?

d) Is there a least element?

e) Find all upper bounds of \{3, 5\}.

f) Find the least upper bound of \{3, 5\}, if it exists.

g) Find all lower bounds of \{15, 45\}.

h) Find the greatest lower bound of \{15, 45\}, if it exists.

9.6 pg. 632 # 43

Determine whether the posets with these Hasse diagrams are lattices.

a)
9.6 pg. 633 # 67

Find an ordering of the tasks of a software project if the Hasse diagram for the tasks of the project is shown.