9.3 Representing Relations

9.3 pg. 596 # 1
Represent each of these relations on \( \{1, 2, 3\} \) with a matrix (with the elements of this set listed in increasing order).

a \{ (1, 1), (1, 2), (1, 3) \}
b \{ (1, 2), (2, 1), (2, 2), (3, 3) \}
c \{ (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3) \}
d \{ (1, 3), (3, 1) \}

9.3 pg. 596 # 3
List the ordered pairs in the relations on \( \{1, 2, 3\} \) corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a \[
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 1 \\
\end{bmatrix}
\]
b \[
\begin{bmatrix}
0 & 1 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0 \\
\end{bmatrix}
\]
c \[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}
\]

9.3 pg. 596 # 9
How many nonzero entries does the matrix representing the relation \( R \) on \( A = \{1, 2, 3, \ldots, 100\} \) consisting of the first 100 positive integers if \( R \) is

a \{ (a, b) | a > b \}?
b \{ (a, b) | a \neq b \}?
c \{ (a, b) | a = b + 1 \}?
d \{ (a, b) | a = 1 \}?
e \{ (a, b) | ab = 1 \}?
9.3 pg. 596 # 13

Let $R$ be the relation represented by the matrix

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Find the matrix representing

a) $R^{-1}$

b) $\overline{R}$

c) $R^2$

9.3 pg. 597 # 19

Draw the directed graphs representing each of the relations

a) $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$

b) $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$

c) $\{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$

d) $\{(2, 4), (3, 1), (3, 2), (3, 4)\}$

9.3 pg. 597 # 23

List the ordered pairs in the relations represented by the directed graph.

![Directed Graph](image)

9.3 pg. 597 # 25

List the ordered pairs in the relations represented by the directed graph.

![Directed Graph](image)
9.3 pg. 597 # 31

Determine whether the relation represented by the digraph shown in Exercises 23 and 25 are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.