5.5 Program Correctness

A program, or program segment, S is said to be partially correct with respect to the initial assertion p and the final assertion q if whenever p is true for the input values of S and S terminates, then q is true for the output values of S. The notation $p\{S\}q$ indicates that the program, or program segment, S is partially correct with respect to the initial assertion p and the final assertion q.

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Prove that the program segment

```
y := 1
z := x + y
```

is correct with respect to the initial assertion $x = 0$ and the final assertion $z = 1$.

Suppose that $x = 0$ as per our initial assertion. First, the program will assign $y = 1$. Next, $x + y$ is computed and assigned to z. Since $x + y$ is $0 + 1$, the result is $z = 1$. Because $z = 1$, the final assertion is satisfied.

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Verify that the program segment

```
x := 2
z := x + y
if y > 0 then
   z := z + 1
else
   z := 0
```

is correct with respect to the initial assertion $y = 3$ and the final assertion $z = 6$.

Suppose that $y = 3$ as per our initial assertion. First, the program will assign $x = 2$. Next, $x + y$ is computed and assigned to z. Since $x + y$ is $2 + 3$, the result is $z = 5$. The program will now check the conditional statement $y > 0$ for the if...else block. $3 > 0$, so $z := z + 1$ is executed and the else clause is ignored. Therefore, $z = 5 + 1 = 6$ and our final assertion is satisfied.

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Use a loop invariant to prove that the following program segment for computing the nth power, n is a positive integer, of a real number x is correct.

```
power := 1
i := 1
while i ≤ n
   power := power · x
   i := i + 1
```

Let p be the loop invariant where "$power = x^{i-1}$ and $i \leq n + 1$." We must show that p is true before the loop, after an iteration of the loop, and after termination of the loop.

- Before the loop, $i = 1$ and $power = x^{1-1} = x^0 = 1$.
- Entering the loop, we know that $i \leq n$. Since i is incremented by 1, we also know that $i \leq n + 1$ after an iteration of the loop. As for $power$, we know that $power_{initial} = x^{i-1}$. The new value for $power$ is $power_{\text{new}} = power_{\text{initial}} \cdot x = x^{i-1} \cdot x = x^i = x^{(i+1)-1}$. However, because $i = i + 1$ from before, $power_{\text{new}} = x^{i-1}$.
- After the termination of the loop, we know that $i \leq n$ is false, so $i = n+1$. Since $i = n+1$, $power = x^{i-1} = x^{n+1-1} = x^n$ as desired.

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Suppose that both the program assertion $p\{S\}q_0$ and the conditional statement $q_0 \to q_1$ are true. Show that $p\{S\}q_1$ also must be true.

Since $p\{S\}q_0$ is true, we know that p is true before S is executed. After S is executed, we know that q_0 is true. Because $q_0 \to q_1$ is true given the problem statement and q_0 is true from before, we can conclude q_1 is true by modus ponens.