5.5 Program Correctness

A program, or program segment, $S$ is said to be partially correct with respect to the initial assertion $p$ and the final assertion $q$ if whenever $p$ is true for the input values of $S$ and $S$ terminates, then $q$ is true for the output values of $S$. The notation $p\{S\}q$ indicates that the program, or program segment, $S$ is partially correct with respect to the initial assertion $p$ and the final assertion $q$.

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Prove that the program segment

\[
\begin{align*}
y &:= 1 \\
z &:= x + y
\end{align*}
\]

is correct with respect to the initial assertion $x = 0$ and the final assertion $z = 1$.

Suppose that $x = 0$ as per our initial assertion. First, the program will assign $y = 1$. Next, $x + y$ is computed and assigned to $z$. Since $x + y$ is $0 + 1$, the result is $z = 1$. Because $z = 1$, the final assertion is satisfied.

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Verify that the program segment

\[
\begin{align*}
x &:= 2 \\
z &:= x + y \\
\text{if } y > 0 \text{ then} & \quad z := z + 1 \\
\text{else} & \quad z := 0
\end{align*}
\]

is correct with respect to the initial assertion $y = 3$ and the final assertion $z = 6$.

Suppose that $y = 3$ as per our initial assertion. First, the program will assign $x = 2$. Next, $x + y$ is computed and assigned to $z$. Since $x + y$ is $2 + 3$, the result is $z = 5$. The program will now check the conditional statement $y > 0$ for the if...else block. $3 > 0$, so $z := z + 1$ is executed and the else clause is ignored. Therefore, $z = 5 + 1 = 6$ and our final assertion is satisfied.

5.5 pg. 377 # 7

Use a loop invariant to prove that the following program segment for computing the $n$th power, $n$ is a positive integer, of a real number $x$ is correct.

\[
\begin{align*}
power &:= 1 \\
i &:= 1 \\
\text{while } i \leq n & \quad \begin{align*}
power &:= power \cdot x \\
i &:= i + 1
\end{align*}
\end{align*}
\]

Let $p$ be the loop invariant where “$\text{power} = x^{i-1}$ and $i \leq n + 1$.” We must show that $p$ is true before the loop, after an iteration of the loop, and after termination of the loop.
• Before the loop, $i = 1$ and $\text{power} = x^{i-1} = x^0 = 1$.

• Entering the loop, we know that $i \leq n$. Since $i$ is incremented by 1, we also know that $i \leq n + 1$ after an iteration of the loop. As for $\text{power}$, we know that $\text{power}_{\text{initial}} = x^{i-1}$. The new value for $\text{power}$ is $\text{power}_{\text{new}} = \text{power}_{\text{initial}} \cdot x = x^{i-1} \cdot x = x^i = x^{(i+1)-1}$. However, because $i = i + 1$ from before, $\text{power}_{\text{new}} = x^{i-1}$.

• After the termination of the loop, we know that $i \leq n$ is false, so $i = n + 1$. Since $i = n + 1$, $\text{power} = x^{i-1} = x^{n+1-1} = x^n$ as desired.

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Suppose that both the program assertion $p\{S\}q_0$ and the conditional statement $q_0 \rightarrow q_1$ are true. Show that $p\{S\}q_1$ also must be true.

Since $p\{S\}q_0$ is true, we know that $p$ is true before $S$ is executed. After $S$ is executed, we know that $q_0$ is true. Because $q_0 \rightarrow q_1$ is true given the problem statement and $q_0$ is true from before, we can conclude $q_1$ is true by modus ponens.