4.2 Integer Representation

Let b be a positive integer greater than 1. Then if n is a positive integer, it can be expressed uniquely in the form:

$$n = a_k b^k + a_{k-1} b^{k-1} + \ldots + a_1 b + a_0$$

where k is a nonnegative integer, $a_0, a_1, \ldots, a_k$ are nonnegative integers less than b, and $a_k \neq 0$.

Common Bases

- Binary: $b = 2$, digits:{0,1}
- Octal: $b = 8$, digits:{0,1,2,3,4,5,6,7}
- Decimal: $b = 10$, digits:{0,1,2,3,4,5,6,7,8,9}
- Hexadecimal: $b = 16$, digits:{0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F}

Convert Decimal n to base b Expansion

1. Divide n by b.
2. The remainder is the rightmost digit in base b expansion of n
3. The quotient becomes the new dividend
4. Stop if quotient is zero
5. Repeat

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Convert the binary expansion of each of these integers to a decimal expansion.

a) $(1\ 1111)_2$
$1 \cdot 2^4 + 1 \cdot 1 \cdot 2^3 + 1 \cdot 1 \cdot 2^2 + 1 \cdot 1 \cdot 2^1 + 1 \cdot 1 \cdot 2^0$
$= 16 + 8 + 4 + 2 + 1 = 31$

c) $(1\ 0101\ 0101)_2$
$1 \cdot 2^8 + 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$
$= 256 + 0 + 64 + 0 + 16 + 0 + 4 + 0 + 1 = 341$

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Convert the decimal expansion of these integers to a binary expansion.

a) 231

$231 = 2 \cdot 115 + 1$
$115 = 2 \cdot 57 + 1$
$57 = 2 \cdot 28 + 1$
$28 = 2 \cdot 14 + 0$
$14 = 2 \cdot 7 + 0$
$7 = 2 \cdot 3 + 1$
$3 = 2 \cdot 1 + 1$
$1 = 2 \cdot 0 + 1$
$231 = (1110\ 0111)_2$

b) 4532

$4532 = 2 \cdot 2266 + 0$
$2266 = 2 \cdot 1133 + 0$
$1133 = 2 \cdot 566 + 1$
$566 = 2 \cdot 283 + 0$
$283 = 2 \cdot 141 + 1$
$141 = 2 \cdot 70 + 1$
$70 = 2 \cdot 35 + 0$
$35 = 2 \cdot 17 + 1$
$17 = 2 \cdot 8 + 1$
$8 = 2 \cdot 4 + 0$
$4 = 2 \cdot 2 + 0$
$2 = 2 \cdot 1 + 0$
$1 = 2 \cdot 0 + 1$
$4532 = (1\ 0001\ 1011\ 0100)_2$

Convert Binary to Octal

Group binary bits into groups of 3 (starting from the right)
Translate each group from a binary group to an octal digit (between 0 and 7 inclusive)

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Convert $(10\ 1011\ 1011)_2$ to its octal expansion

001	010	111	011
1	2	7	3

$(10\ 1011\ 1011)_2 = (1273)_8$

Convert Octal to Binary

Starting from the right most octal digit
Translate each octal digit to a binary string of length 3

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Convert the octal expansion of each of these integers to a binary expansion.

a) $(572)_8$

5	7	2
101	111	010

$(572)_8 = (1\ 0111\ 1010)_2$

Convert Binary to Hexadecimal

Group binary bits into groups of 4 (starting from the right)
Translate each group from a binary group to a hexadecimal digit (between 0 and F inclusive)

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Convert $(1011\ 0111\ 1011)_2$ from its binary expansion to its hexadecimal expansion.

1011	0111	1011
B	7	B

$(1011\ 0111\ 1011)_2 = (B7B)_{16}$

Convert Hexadecimal to Binary

Starting from the right most hexadecimal digit
Translate each hexadecimal digit to a binary string of length 4

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Convert $(ABCDEF)_{16}$ from its hexadecimal expansion to its binary expansion.

A	B	C	D	E	F
1010	1011	1100	1101	1110	1111

$(ABCDEF)_{16} = (1010\ 1011\ 1100\ 1101\ 1110\ 1111)_2$

Binary Addition

carry = $\lfloor bitSum/2 \rfloor$
bit_index = $bitSum \mod 2$

Example:

Carry:	1	1		
	0	1	1	0
+	0	1	1	0
	1	1	0	0

Binary Multiplication

Works like normal multiplication.

			1	0	1
		x	1	0	1
			1	0	1
		0	0	0	0
+	1	0	1	0	0
	1	1	0	0	1

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Find the sum and product of each of these pairs of numbers. Express your answers as a binary expansion.

a) $(100\,0111)_2$, $(111\,0111)_2$

Sum:

Carry:					1	1	1	
		1	0	0	0	1	1	1
	+	1	1	1	0	1	1	1
	1	0	1	1	1	1	1	0

Product:

								1	0	0	0	1	1	1
							x	1	1	1	0	1	1	1
Carry:		1	1	1	2	3	3	2	2	2	2	1		
								1	0	0	0	1	1	1
							1	0	0	0	1	1	1	0
						1	0	0	0	1	1	1	0	0
					0	0	0	0	0	0	0	0	0	0
				1	0	0	0	1	1	1	0	0	0	0
			1	0	0	0	1	1	1	0	0	0	0	0
	+	1	0	0	0	1	1	1	0	0	0	0	0	0
	1	0	0	0	0	1	0	0	0	0	0	0	0	1