1.4 Predicates and Quantifiers

1.4 pg. 53 # 5

Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x P(x)$

b) $\forall x P(x)$

c) $\exists x \neg P(x)$

d) $\forall x \neg P(x)$

1.4 pg. 53 # 11

Let $P(x)$ be the statement "$x = x^2$." If the domain consists of all the integers, what are these truth values?

a) $P(0)$

c $P(2)$

e $\exists x P(x)$

f $\forall x P(x)$

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Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives. The domain of x is all people.

c All your friends are perfect.

d At least one of your friends is perfect.

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Express each of these statements using quantifiers. Then form the negation of the statement, so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

b No rabbit knows calculus.

c Every bird can fly.

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Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a $\forall x(x^2 \geq x)$

b $\forall x(x > 0 \lor x < 0)$

c $\forall x(x = 1)$

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Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a professor," "x is ignorant," and "x is vain," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$, where the domain consists of all people.

a No professors are ignorant.

b All ignorant people are vain.

c No professors are vain.

d Does (c) follow from (a) and (b)?