5.3 Recursive Definitions

5.3 pg 357 # 1

Find $f(1), f(2), f(3)$, and, $f(4)$ if $f(n)$ is defined recursively by $f(0) = 1$ and for $n = 0, 1, 2, \ldots$

a) $f(n + 1) = f(n) + 2$

b) $f(n + 1) = 3f(n)$

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Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \ldots$ if

a) $a_n = 6n$

b) $a_n = 2n + 1$

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Give a recursive definition of

a) the set of even integers.

b) the set of positive integers congruent to 2 modulo 3.

c) the set of positive integers not divisible by 5.

5.3 pg 358 # 27

Let S be the subset of the set of ordered pairs of integers defined recursively by

- Basis Step: $(0, 0) \in S$
- Recursive Step: If $(a, b) \in S$, then $(a, b+1) \in S$, $(a+1, b+1) \in S$, and $(a+2, b+1) \in S$.

a) List the elements of S produced by the first four applications of the recursive definition.

c) Use structural induction to show that $a \leq 2b$ whenever $(a, b) \in S$.

5.3 pg 359 # 37

Give a recursive definition of w^i, where w is a string and i is a nonnegative integer. (Here w^i represents the concatenation of i copies of the string w.)