2.2 Set Operations

Union

The *union* of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both. $A \cup B = \{x \mid x \in A \vee x \in B\}$

Intersection

The *intersection* of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B. $A \cap B = \{x \mid x \in A \wedge x \in B\}$.

Disjoint

Two sets A, B are said to be *disjoint* if and only if their intersection is empty. $A \cap B = \emptyset$.

Complement

The complement of A (with respect to U), denoted by $\overline{A}$, is the set of elements not in A. (i.e. $U - A$). $\overline{A} = \{x \mid x \notin A\}$

Set Difference

The difference of A and B, denoted by $A - B$, is the set containing the elements of A that are not in B. $A - B = \{x \mid x \in A \wedge x \notin B\} = A \cap \overline{B}$

Set Identities

TABLE 1 Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

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Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

a $A \cup B$

$\{0, 1, 2, 3, 4, 5, 6\}$

b $A \cap B$

$\{3\}$

c $A - B$

$\{1, 2, 4, 5\}$

d $B - A$

$\{0, 6\}$

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Prove the second De Morgan law in Table 1 by showing that if A and B are sets, then $\overline{A \cup B} = \overline{A} \cap \overline{B}$

a by showing each side is a subset of the other side.

$\overline{A \cup B}$

$= \{x \mid x \notin A \cup B\}$	by definition of compliment
$= \{x \mid \neg(x \in (A \cup B))\}$	by definition of does not belong symbol
$= \{x \mid \neg(x \in A \vee x \in B)\}$	by definition of union
$= \{x \mid \neg(x \in A) \wedge \neg(x \in B)\}$	by De Morgan's law (for logical equivalence)
$= \{x \mid x \notin A \wedge x \notin B\}$	by definition of does not belong symbol
$= \{x \mid x \in \overline{A} \wedge x \in \overline{B}\}$	by definition of complement
$= \{x \mid x \in \overline{A} \cap \overline{B}\}$	by definition of intersection
$= \overline{A} \cap \overline{B}$	by set builder notation

b using a membership table

A	B	$\overline{A}$	$\overline{B}$	$A \cup B$	$\overline{A \cup B}$	$\overline{A} \cap \overline{B}$
1	1	0	0	1	0	0
1	0	0	1	1	0	0
0	1	1	0	1	0	0
0	0	1	1	0	1	1

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Show that if A and$B$ are sets, then

a $A - B = A \cap \overline{B}$

Both represent $\{x | x \in A \wedge x \notin B\}$

b $(A \cap B) \cup (A \cap \overline{B}) = A$

$(A \cap B) \cup (A \cap \overline{B})$

$= A \cap (B \cup \overline{B})$	by distributive law
$= A \cap U$	by complement law
$= A$	by identity law

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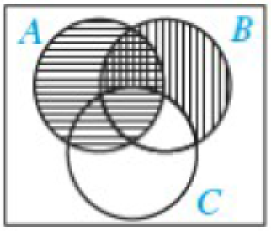
Draw the Venn diagrams for each of these combinations of the sets A, B, C.

a $A \cap (B - C)$

The double shaded portion is the desired set.



b $(A \cap B) \cup (A \cap C)$

The entire shaded portion is the desired set.