2.1 Sets

A *set* is an unordered collection of objects, called *elements* or *members* of the set. A *set* is said to contain its elements.

We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

Two sets are equal if and only if they have the same elements. We write $A = B$ if A and B are equal sets.

Empty Set

The *empty set* or *null set* is the set with no elements. Denoted by $\emptyset$ or $\{\}$.

Other Special Sets

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$, the set of natural numbers

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of integers

$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$, the set of positive integers

$\mathbb{Q} = \{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}$, and $q \notin 0\}$, the set of rational numbers

$\mathbb{R}$, the set of real numbers

$\mathbb{R}^+$, the set of positive real numbers

$\mathbb{C}$, the set of complex numbers.

Subset

The set A is a subset of B if and only if every element of A is also an element of B. We use the notation $A \subseteq B$ to indicate that A is a subset of the set B.

To show that $A \nsubseteq B$, find a single $x \in A$ such that $x \notin B$.

Note that $\emptyset$ is the subset of every set.

Proper Subset

To show that A is a subset of B and $A \neq B$, we use $\subset$ to denote *proper subset*. $A \subset B$ says that A is a proper subset of B.

Cardinality

The *cardinality* of a set is the number of distinct elements within the set. The cardinality of set A is $|A|$. Note that $|\emptyset| = 0$.

Power Sets

The set of all subsets of a set A, denoted $\rho(A)$, is called the *power set* of A.

Cartesian Product

The Cartesian Product of two sets A and B, denoted by $A \times B$, is the set of ordered pairs (a, b) where $a \in A$ and $b \in B$.

Note that $A \times B \neq B \times A$.

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List the members of these sets.

c $\{x \mid x \text{ is the square of an integer and } x < 100\}$

$\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

d $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

$\emptyset$ or $\{\}$

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Determine whether each pairs of sets are equal.

a $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$

Yes

b $\{\{1\}\}, \{1, \{1\}\}$

No

c $\emptyset, \{\emptyset\}$

No

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Determine whether each of these statements is true or false.

a $0 \in \emptyset$

False

b $\emptyset \in \{0\}$

False

d $\emptyset \subset \{0\}$

True

e $\{0\} \in \{0\}$
False

f $\{0\} \subset \{0\}$
False

g $\{\emptyset\} \subseteq \{\emptyset\}$
True

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Determine whether each of these statements is true or false.

a $x \in \{x\}$
True

b $\{x\} \subseteq \{x\}$
True

c $\{x\} \in \{x\}$
False

d $\{x\} \in \{\{x\}\}$
True

e $\emptyset \subseteq \{x\}$
True

f $\emptyset \in \{x\}$
False

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What is the cardinality of each of of these sets?

b $\{\{a\}\}$
1

c $\{a, \{a\}\}$
2

d $\{a, \{a\}, \{a, \{a\}\}\}$
3

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Find the power set of each of these sets, where a and b are distinct elements.

a $\{a\}$
$\{\emptyset, \{a\}\}$

b $\{a, b\}$
$\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

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Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same.

First, $A \times B \times C$ consists of 3-tuples (a, b, c), where $a \in A$, $b \in B$, and $c \in C$. Next, $(A \times B) \times C$ contains the elements $((a, b), c)$, which is a set of ordered pairs with one of them being an ordered pair. An ordered pair and a 3-tuple are two different collections.