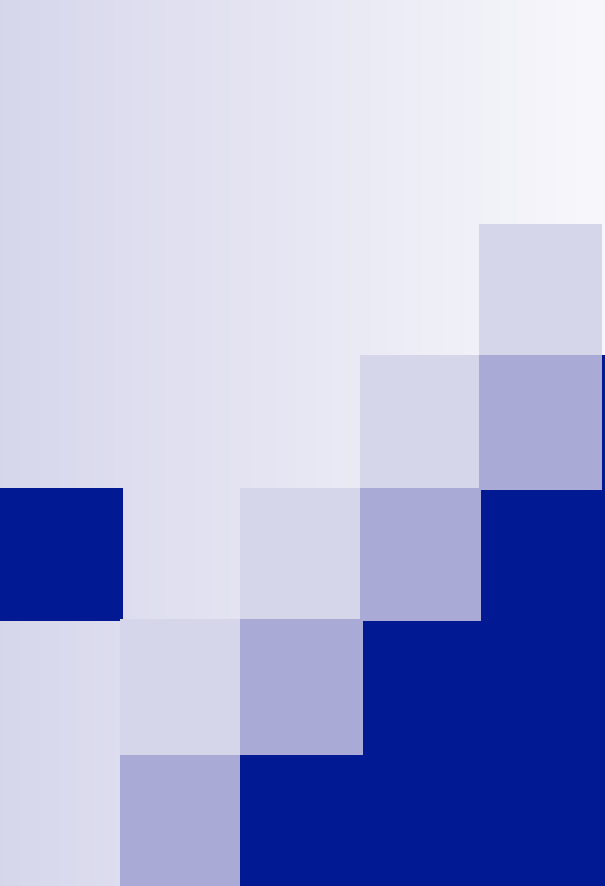




Background/Review on Integers and Bases (practice)

ICS312 Machine-Level and Systems Programming

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Outline

■ Conversions

- Of course, online tools will do it all for us easily
- And we have systematic algorithms
- The point here is to understand the structure of the numbers, to be able to think quickly in terms of powers of 2
- The point isn't to apply the systematic algorithm

■ Additions

- Here again, online tools will do it all
- The point is to understand how carries propagate, which comes in handy to understand various techniques and algorithms

■ Unless you ask we are going to skip most, and you can do them on your own

- Let's only do q2 and q3 "live"



(q1) Decimal to Binary

- What is 52_{10} in binary?

(q1) Solution

- What is 52_{10} in binary?
- Systematic method (which we don't really care about since that's what online tools will do):
 - $52 = 26 \cdot 2 + 0$
 - $26 = 13 \cdot 2 + 0$
 - $13 = 6 \cdot 2 + 1$
 - $6 = 3 \cdot 2 + 0$
 - $3 = 1 \cdot 2 + 1$
 - $1 = 0 \cdot 2 + 1$
 - Answer: 110100
- Intuitive method (for “small” numbers, the computer scientist should think of it like this)
 - 52 is lower than 64, so it's 32 + something
 - $32 + 16$ is 48, so 52 is $32 + 16 + \text{something}$
 - $52 - 48 = 4$, so: $52 = 32 + 16 + 4$
 - Therefore: 110100
 - We have 32, 16, not 8, 4, not 2, not 1



(q2) Decimal to Binary

- What is 2049_{10} in binary?

(q2) Solution

- What is 2049_{10} in binary?
- The systematic method is really long here
- It's easier to see that 2049_{10} is $2048_{10} + 1_{10}$
 - 2048_{10} is $2^{11} = 1000000000000_2$
 - 1 is $2^0 = 1_2$
- Therefore
 - $2049_{10} = 1000000000001_2$
- Again, we find “nearby” powers of 2 (as computer scientists, we know powers of 2)



(q3) Decimal to Binary

- What is 1021_{10} in binary?

(q3) Solution

- What is 1021_{10} in binary?
- 1021_{10} is “close to” 1024_{10}
- We know that 1024_{10} is 100000000000_2
 - A 1 followed by 10 0's
- And now we can count backwards:
 - 1023_{10} is 1111111111_2
 - 1022_{10} is 1111111110_2
 - 1021_{10} is 1111111101_2
- Answer: 1111111101_2

(q4) Hex to Binary

- What is $B8_{16}$ in binary?

(q4) Solution

- Just “glue” the 2 4-bit conversions together
 - $B_{16} = 1011_2$, $8_{16} = 1000_2$
 - Answer: 10111000

- How do I know that $B_{16} = 1011_2$?
 - Just go back to decimal
 - $B_{16} = 11_{10}$
 - $11_{10} = 1011_2$

(q5) Decimal to Hex

- What is 51_{10} in hexadecimal?

(q5) Solution

- What is 51_{10} in hexadecimal?
 - $51 = 3 \cdot 16 + 3$
 - Answer: 33



(q6) Binary to Hex

- What is 100110_2 in hexadecimal?

(q6) Solution

- What is 100110_2 in hexadecimal?
 - 0010 0110
 - $0010_2 = 2_{10} = 2_{16}$
 - $0110_2 = 6_{10} = 6_{16}$
 - Answer: 26

More Practice

- (q7) What is 123_{10} in binary?
- (q8) What is $F3EA_{16}$ in binary?
- (q9) What is 111_{10} in hexadecimal?
- (q10) What is 100110_2 in hexadecimal?

Solutions

■ Conversions:

- (q7) What is 123_{10} in binary?

1111011 (127 - 4)

- (q8) What is $F3EA_{16}$ in binary?

1111001111101010

- (q9) What is 111_{10} in hexadecimal?

6F (112 - 1)

- (q10) What is 100110_2 in hexadecimal?

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Always try to find simple “tricks” if you can



Outline

■ Conversions

- Of course, online tools will do it all for us easily
- And we have systematic algorithms
- The point here is to understand the structure of the numbers, to be able to think quickly in terms of powers of 2
- The point isn't to apply the systematic algorithm

■ Additions

- Here again, online tools will do it all
- The point is to understand the structure of the numbers, how carry propagate, etc.

(q11) Binary addition

- Is the result of $10101100 + 11001011$ odd or even?
- What about $101010100 + 0001010110$?

(q11) Solution

- Is the result of $10101100 + 11001011$ odd or even?
 - even + odd $\rightarrow$ odd
 - The least significant bit of the result is a 1
- What about $101010100 + 0001010110$?
 - even + even $\rightarrow$ even
 - The least significant bit of the result is a 0



(q12) Binary addition

- What is: $10101101 + 11001011$?

(q12) Solution

- What is: $10101101 + 11001011$?

$$\begin{array}{r} \text{c} \quad \text{cccc} \\ 10101101 \\ + \quad 11001011 \\ = \quad 101111000 \end{array}$$



(q13) Hex addition

- What is: $A5F + E32$?

(q13) Solution

- What is: $A5F + E32$?

$$\begin{array}{r} \text{c} \quad \text{c} \\ A5F \\ + \quad E32 \\ = \quad 1891 \end{array}$$

- Small “trick”: adding F to a digit takes that digit 1 lower and generates a carry
 - $F + 7 = 6$ and a carry
 - $F + E = D$ and a carry

More practice

- (q14) What is $1010111 + 1110111$?
- (q15) What is $AF3F + EE8D$?

More practice

- (q14) What is $1010111 + 1110111$?

$$\begin{array}{r} \text{ccc} \quad \text{ccc} \\ 1010111 \\ + \quad 1110111 \\ = \quad 11001110 \end{array}$$

- (q15) What is $AF3F + EE8D$?

$$\begin{array}{r} \text{c} \quad \text{c} \\ AF3F \\ + \quad EE8D \\ = \quad 19DCC \end{array}$$