Background/Review on Integers and Bases (practice)

ICS312

Machine-Level and Systems Programming

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Outline

- Conversions
  - Of course, online tools will do it all for us easily
  - And we have systematic algorithms
  - The point here is to understand the structure of the numbers, to be able to think quickly in terms of powers of 2
  - The point isn't to apply the systematic algorithm
- Additions
  - Here again, online tools will do it all
  - The point is to understand how carries propagate, which comes in handy to understand various techniques and algorithms

(q1) Decimal to Binary

- What is 52_{10} in binary?

(q1) Solution

- What is 52_{10} in binary?
- Systematic method (which we don't really care about since that's what online tools will do):
  - 52 = 26*2 + 0
  - 26 = 13*2 + 0
  - 13 = 6*2 + 1
  - 6 = 3*2 + 0
  - 3 = 1*2 + 1
  - 1 = 0*2 + 1
  - Answer: 110100
- Intuitive method (for "small" numbers, the computer scientist should think of it like this)
  - 52 is lower than 64, so it's 32 + something
  - 32 + 16 is 48, so 52 is 32 + 16 + something
  - 52 - 48 = 4, so: 52 = 32 + 16 + 4
  - Therefore: 110100
    - We have 32, 16, not 8, 4, not 2, not 1

(q2) Decimal to Binary

- What is 2049_{10} in binary?

(q2) Solution

- What is 2049_{10} in binary?
- The systematic method is really long here
- It's easier to see that 2049_{10} is $2048_{10} + 1_{10}$
  - 2048_{10} is $2^{11} = 100000000000_2$
  - 1 is $2^0 = 1_2$
- Therefore
  - $2049_{10} = 100000000001_2$

- Again, we find "nearby" powers of 2 (as computer scientists, we know powers of 2)

(q3) Decimal to Binary

- What is 1021_{10} in binary?

(q3) Solution

- What is 1021_{10} in binary?
- 1021_{10} is "close to" 1024_{10}
- We know that 1024_{10} is 10000000000_2
  - A 1 followed by 10 0's
- And now we can count backwards:
  - 1023_{10} is 1111111111_2
  - 1022_{10} is 1111111110_2
  - 1021_{10} is 1111111101_2
- Answer: 1111111101_2

(q4) Hex to Binary

- What is $B8_{16}$ in binary?

(q4) Solution

- Just “glue” the 2 4-bit conversions together
  - $B_{16} = 1011_2$, $8_{16} = 1000_2$
  - Answer: 10111000

- How do I know that $B_{16} = 1011_2$?
  - Just go back to decimal
    - $B_{16} = 11_{10}$
    - $11_{10} = 1011_2$

(q5) Decimal to Hex

- What is 51_{10} in hexadecimal?

(q5) Solution

- What is 51_{10} in hexadecimal?
  - 51 = 3*16 + 3
  - Answer: 33

(q6) Binary to Hex

- What is 100110_2 in hexadecimal?

(q6) Solution

- What is 100110_2 in hexadecimal?
  - 0010 0110
  - $0010_2 = 2_{10} = 2_{16}$
  - $0110_2 = 6_{10} = 6_{16}$
  - Answer: 26

More Practice

- (q7) What is 123_{10} in binary?
- (q8) What is $F3EA_{16}$ in binary?
- (q9) What is 111_{10} in hexadecimal?
- (q10) What is 100110_{2} in hexadecimal?

Solutions

- Conversions:
  - (q7) What is 123_{10} in binary?

    1111011 (127 - 4)
  - (q8) What is $F3EA_{16}$ in binary?

    1111001111101010
  - (q9) What is 111_{10} in hexadecimal?

    6F (112 -1)
  - (q10) What is 100110_2 in hexadecimal?

    26

  Always try to find simple “tricks” if you can

Outline

- Conversions
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  - The point isn't to apply the systematic algorithm
- Additions
  - Here again, online tools will do it all
  - The point is to understand the structure of the numbers, how carry propagate, etc.

(q11) Binary addition

- Is the result of 10101100 + 11001011 odd or even?

- What about 1010101100 + 0001010110?

(q11) Solution

- Is the result of 10101100 + 11001011 odd or even?
  - even + odd → odd
  - The least significant bit of the result is a 1
- What about 101010100 + 0001010110?
  - even + even → even
  - The least significant bit of the result is a 0

(q12) Binary addition

- What is: 10101101 + 11001011 ?

(q12) Solution

- What is: 10101101 + 11001011 ?

```
    c    cccc
     10101101
+    11001011
=   101111000
```

(q13) Hex addition

- What is: A5F + E32 ?

(q13) Solution

- What is: A5F + E32 ?

```
      c  c
       A5F
+      E32
=     1891
```

- Small “trick”: adding F to a digit takes that digit 1 lower and generates a carry
  - F + 7 = 6 and a carry
  - F + E = D and a carry

More practice

- (q14) What is 1010111 + 1110111?

- (q15) What is AF3F + EE8D?

More practice

- (q14) What is 1010111 + 1110111?

```
   ccc ccc
    1010111
+   1110111
=  11001110
```

- (q15) What is AF3F + EE8D?

```
     c  c
    AF3F
+   EE8D
= 19DCC
```